Eniko T. Enikov, University of Arizona
Estelle Eke, California State University Sacramento

PERSONALIZED EXPERIMENTATION IN CLASSICAL CONTROLS WITH MATLAB REAL-TIME WINDOWS TARGET AND PORTABLE AEROPENDULUM KIT

Outline

- Motivation
- The Aeropendulum Apparatus
- Real-Time vs. Soft Real Time
- Student Design Activities
  - Plant modeling,
  - parameter identification, identification of non-linearities
  - Feedback linearization, steady-state error and system types
  - parameter identification
  - Matlab's pem() prediction-error minimization function (time permitting)
  - closed-loop control experiments: proportional, phase lag,
  - phase lead and on/off (bang-bang) control (time permitting)
- Results from implementation at CSUS and Univ. of Arizona

Motivation

- Develop an portable low-cost apparatus that illustrates classical control systems course with a hands-on experimentation.
- Eliminate the need for lab space, teaching assistant.
- Provide a quick pathway from controller design to implementation for mechanical engineering students.

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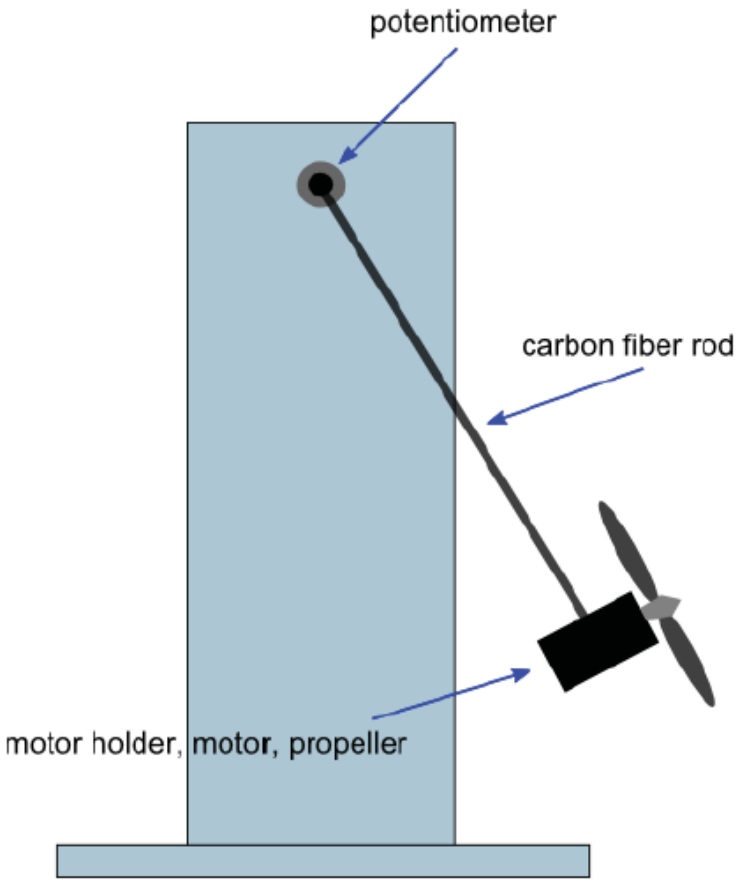
Experimental Apparatus

- Acrylic stand
- Pendulum with angle sensing potentiometer, DC-motor and propeller
- Target circuit board driving the propeller with different PWM ratios in forward and reverse direction
- MATLAB Simulink Real Time Windows Target GUI for controller implementation



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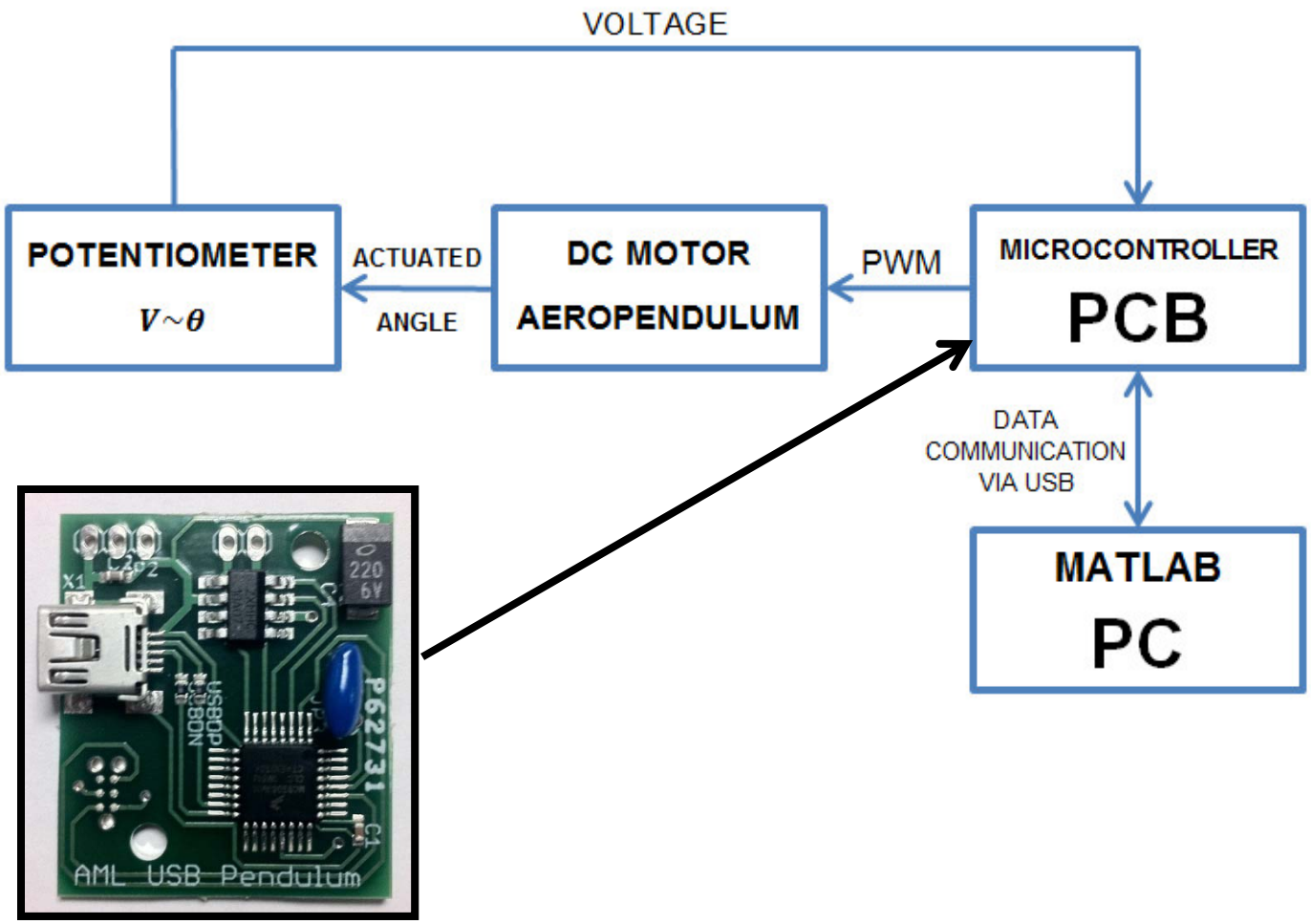
Data Flow Diagram



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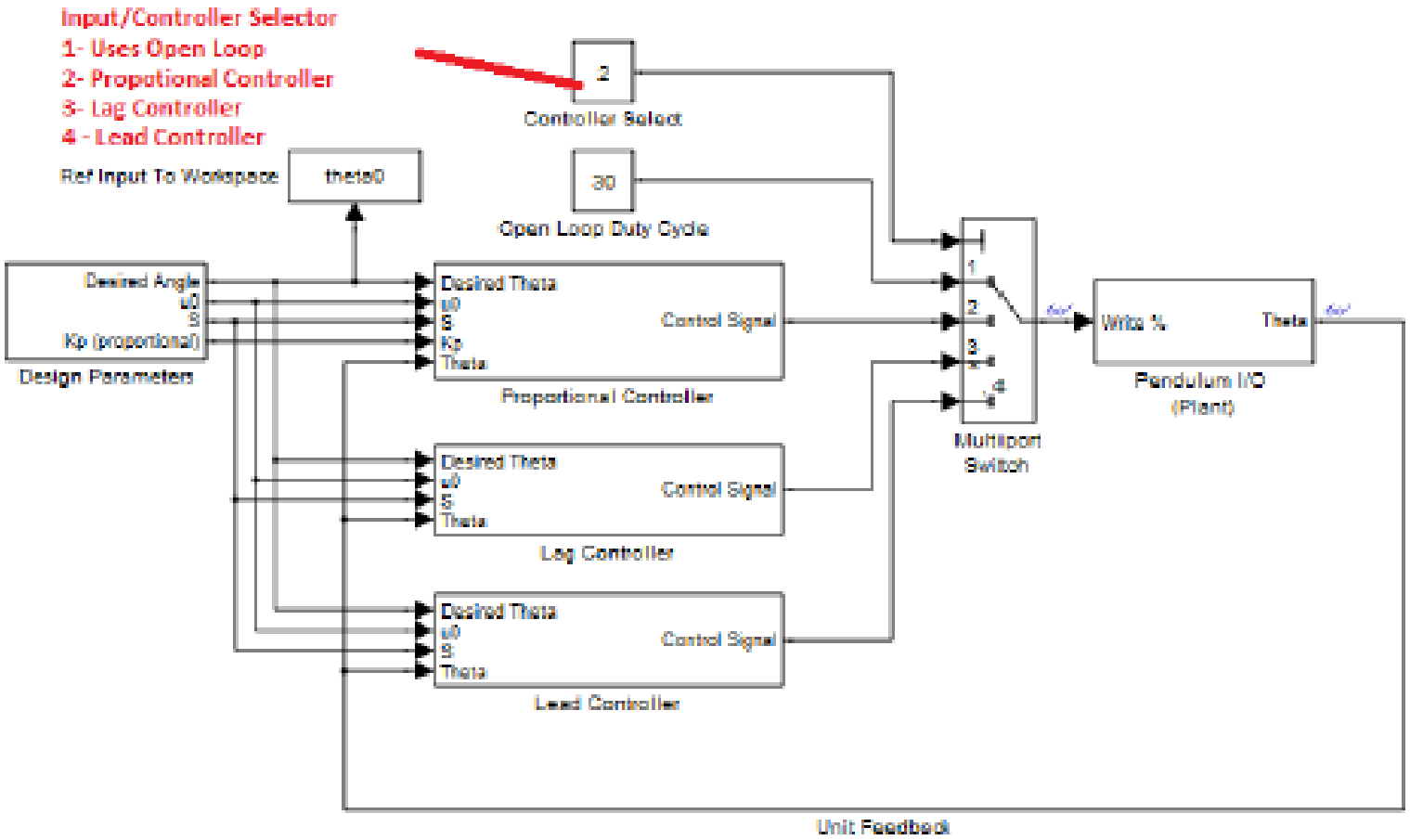
Real Time Windows Target Environment



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Modeling Tasks

$$mL^2\ddot{\theta} = -mgL\sin\theta - c\dot{\theta} + TL$$

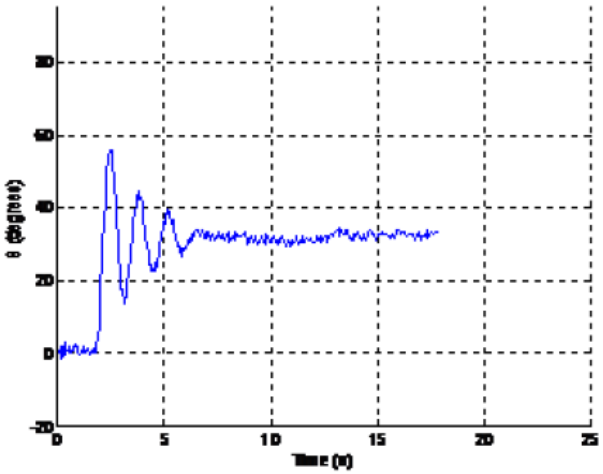
$$T = Ku$$

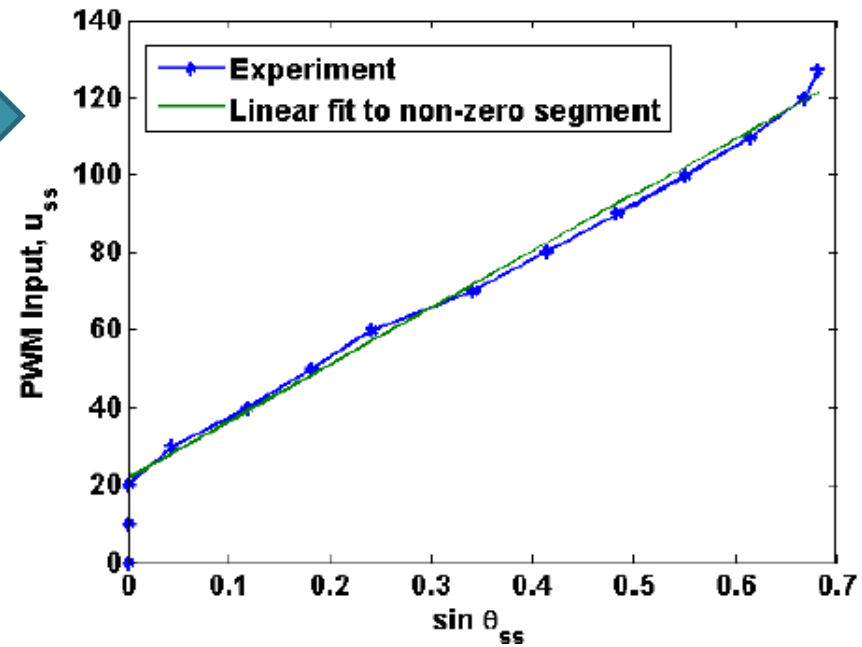
$$mL^2\ddot{\theta} = -mgL\sin\theta - c\dot{\theta} + KLu.$$

Experiment 1: Parameter Extraction

Using the steady-state response, find the parameters K/mg

$$\sin\theta_{ss} = \frac{K}{mg} u_{ss}.$$





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Challenge 1: Dealing with Dead Zone

$$T = \begin{cases} K_+(u - u_0), & \text{if } u > u_0 \\ 0, & \text{if } u \in [-u_0, u_0] \\ -K_-(u + u_0), & \text{if } u < -u_0. \end{cases} \qquad u = \begin{cases} \bar{u} + u_0, & \text{if } \bar{u} > 0 \\ \bar{u} - u_0, & \text{if } \bar{u} < 0, \end{cases}$$

$$mL^2\ddot{\theta} = -mgL\sin\theta - c\dot{\theta} + KL\bar{u}.$$

$$\bar{u} = \frac{mg}{K}\sin\theta + \tilde{u}, \qquad \frac{\Theta(s)}{\tilde{U}(s)} = \frac{KL}{mL^2s^2 + cs}.$$

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Challenge II: Feedback Linearization

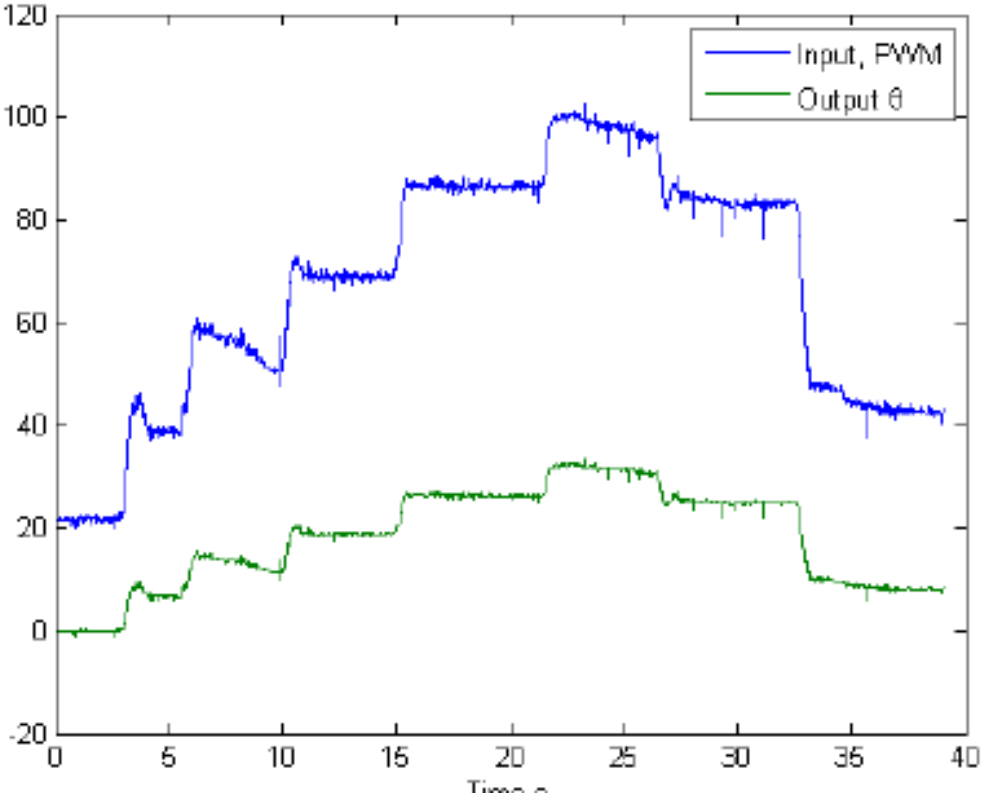
$$mL^2\ddot{\theta} = -mgL\sin\theta - c\dot{\theta} + KL\bar{u}.$$

$$\bar{u} = \frac{mg}{K}\sin\theta + \tilde{u},$$

Result Type 1 System

$$\frac{\Theta(s)}{\tilde{U}(s)} = \frac{KL}{mL^2s^2 + cs}.$$

Experiment II: Weightless Pendulum (K=0)



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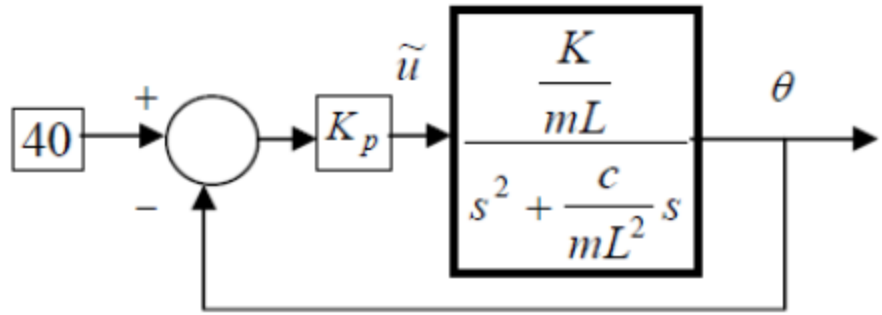
Experiment III: Parameter Identification (Kp=1)

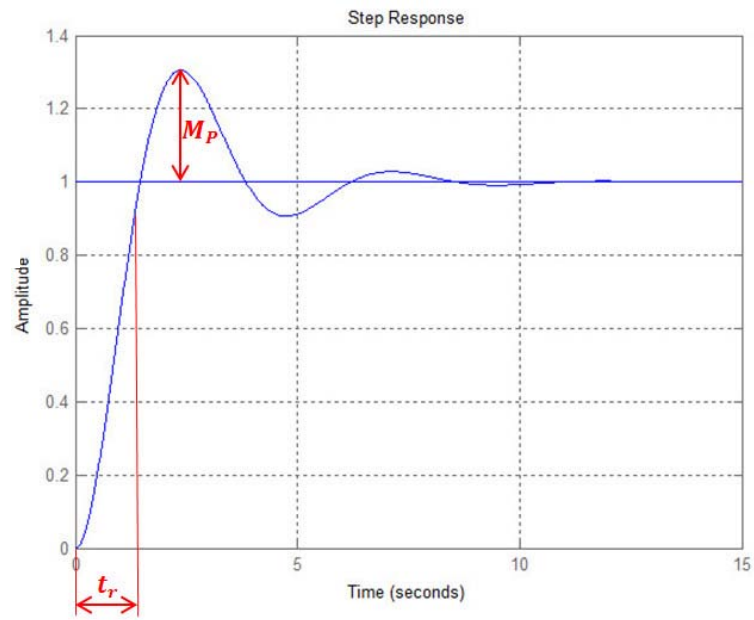


$$P.O. = \exp\left(-\zeta\pi/\sqrt{1-\zeta^2}\right)$$

$$t_{rise} = \frac{1.8}{\omega_n}.$$

$$\omega_n^2 = \frac{K}{mL}$$

$$2\zeta\omega_n = \frac{c}{mL^2}.$$



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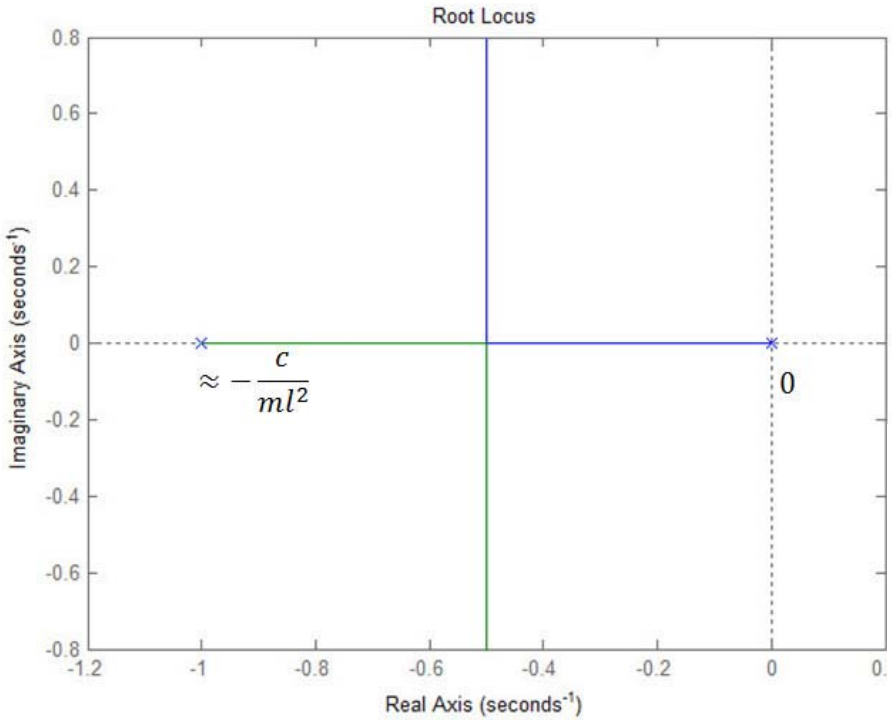
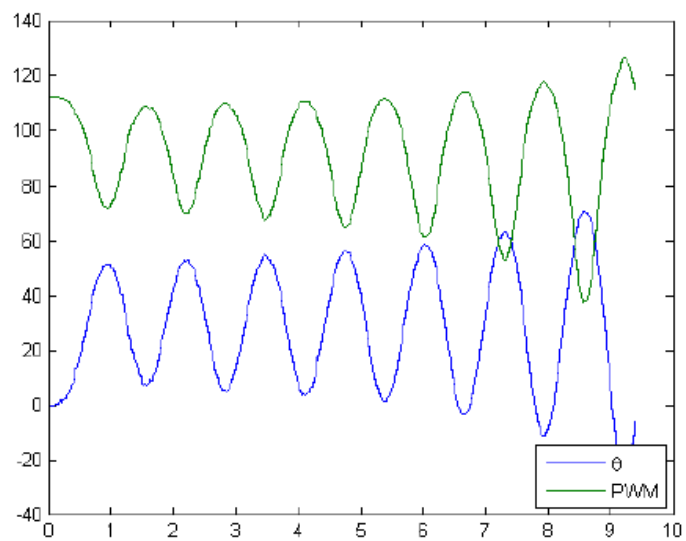
Challenge III: Stability and Root Locus. What is wrong?



Kp>3 (unstable)



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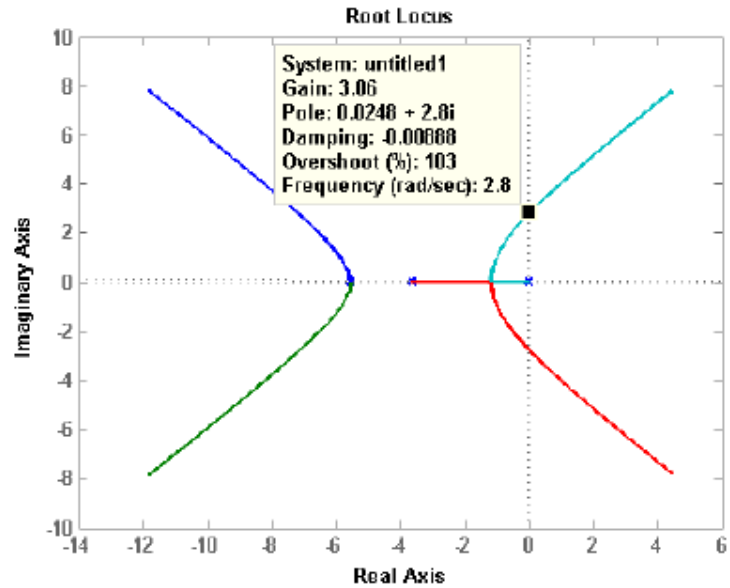
Model Correction: Motor Dynamics

$$V = Ri + K_v\omega + L_{ind}\frac{di}{dt};$$

$$J_m\dot{\omega} = K_T i - C_Q\frac{\rho}{4\pi^2}D^5\omega^2$$

$$\frac{\Theta(s)}{\tilde{U}(s)} = \frac{5.327}{s^2 + 3.649s}\frac{1}{(1+T_D s)^2}$$



$$\frac{\Theta(s)}{\tilde{U}(s)} = \frac{KL}{mL^2s^2 + cs}\frac{1}{(1+T_{D1}s)(1+T_{D2}s)}.$$

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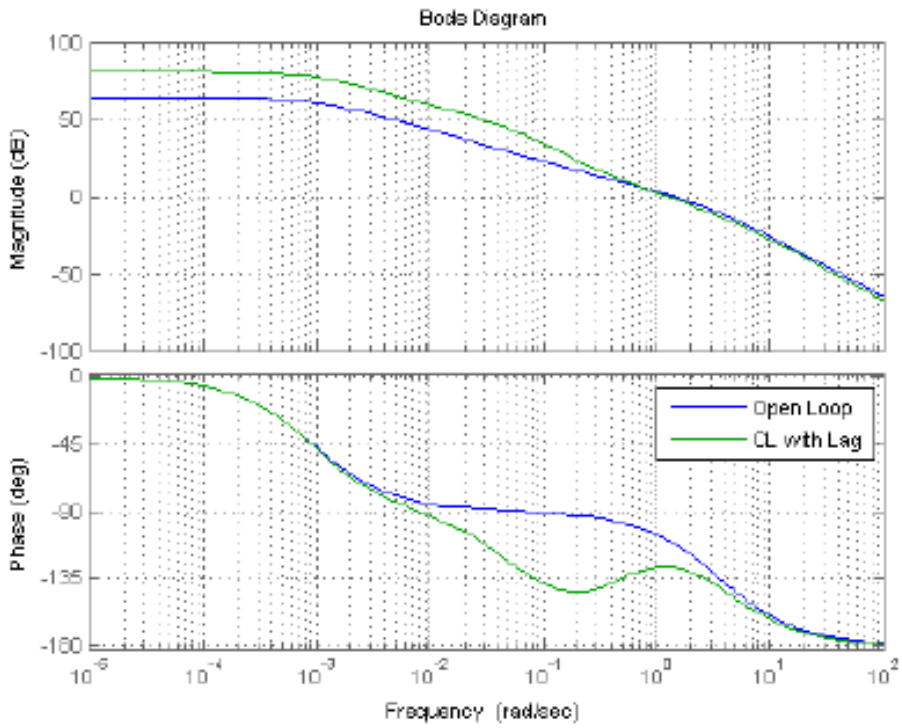
Experiment IV: Controller Design Using Bode Plots



$$\frac{\Theta(s)}{\tilde{U}(s)} = \frac{5.327}{s^2 + 3.649s + 0.0033}.$$

$$C_{lag} = \frac{0.82575(s + 0.4771)}{s + 0.05936}$$

Clagd=c2d(Clag,0.01,'zoh').

$$C_{\text{lagd}} = \frac{0.82575(z - 0.9952)}{z - 0.9994}$$

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Evaluation

STUDENT FEEDBACK

To what extent (how well) did the project illustrate the following technical concepts?

	Not at all	Less than expected	More than expected	Very Well	Rating Average
Relationship between physical system and transfer function	0.0% (0)	7.1%(2)	64.3%(18)	28.6%(8)	3.21
Second-order system response	3.6% (1)	10.7%(3)	60.7%(17)	25.%(7)	3.07
Relationship between stability and gain	0.0% (0)	10.7%(3)	46.4%(13)	42.9%(12)	3.32
Relationship between overshoot and gain	3.6% (1)	25.0%(7)	35.7%(10)	35.7%(10)	3.04
Relationship between overshoot and gain	0.0% (0)	30.8%(8)	42.3%(11)	26.9%(7)	2.96
Use of root locus	0.0% (0)	17.9%(5)	57.1%(16)	25.0%(7)	3.07
Use of Bode plots	14.3% (4)	39.3%(11)	35.7%(10)	10.7%(3)	2.43
System type and steady state error	7.1% (2)	10.7%(3)	50.0%(14)	32.1%(9)	3.07
Disturbance rejection and system recovery	7.4% (2)	11.1%(3)	48.1%(13)	33.3%(9)	3.07
Non-linearities and ways to deal with them	0.0% (0)	14.3%(4)	60.7%(17)	25.0%(7)	3.11
Effects of time delay	0.0% (0)	17.9%(5)	53.6%(15)	28.6%(8)	3.11

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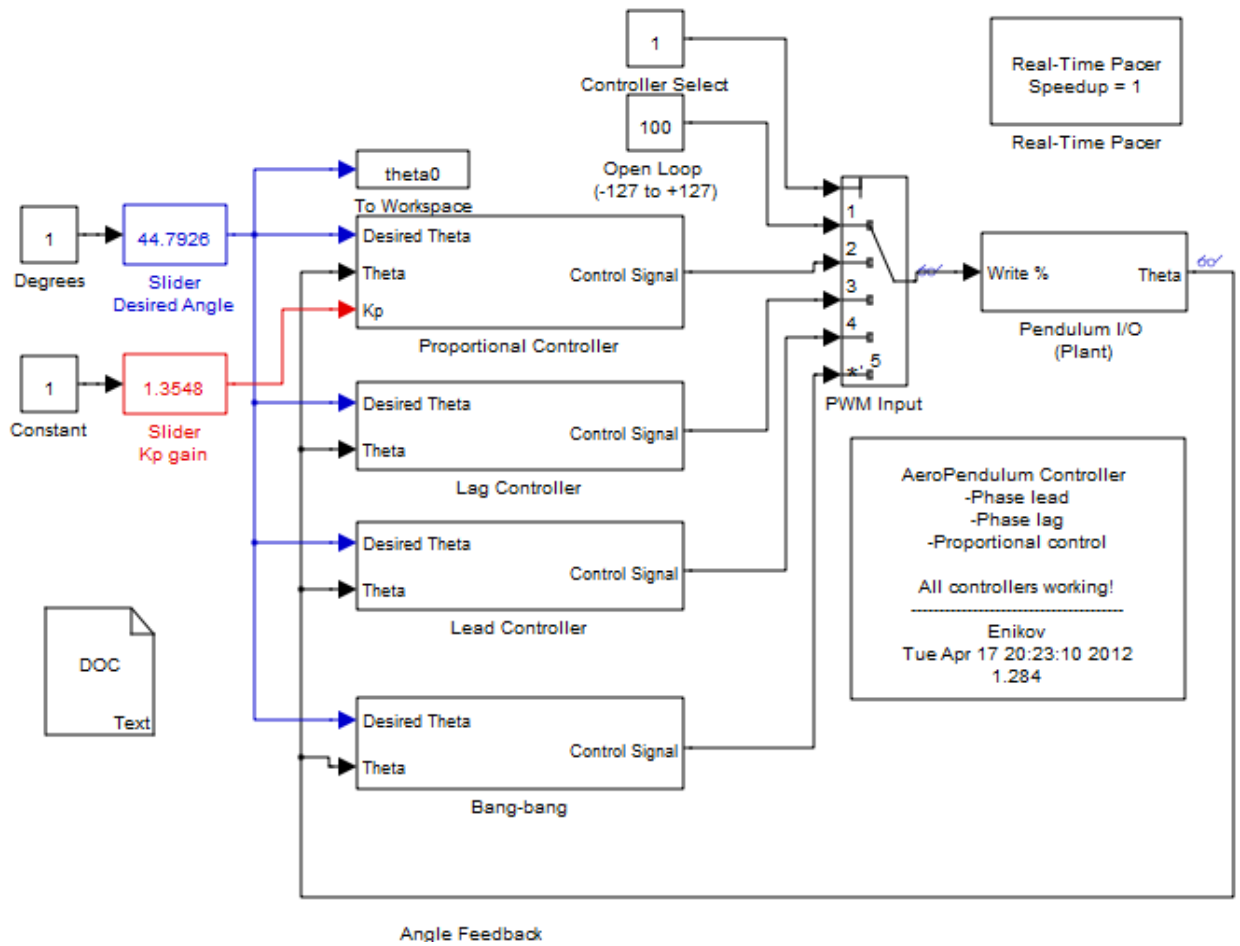
Hands-On Activities

- Open Loop Response



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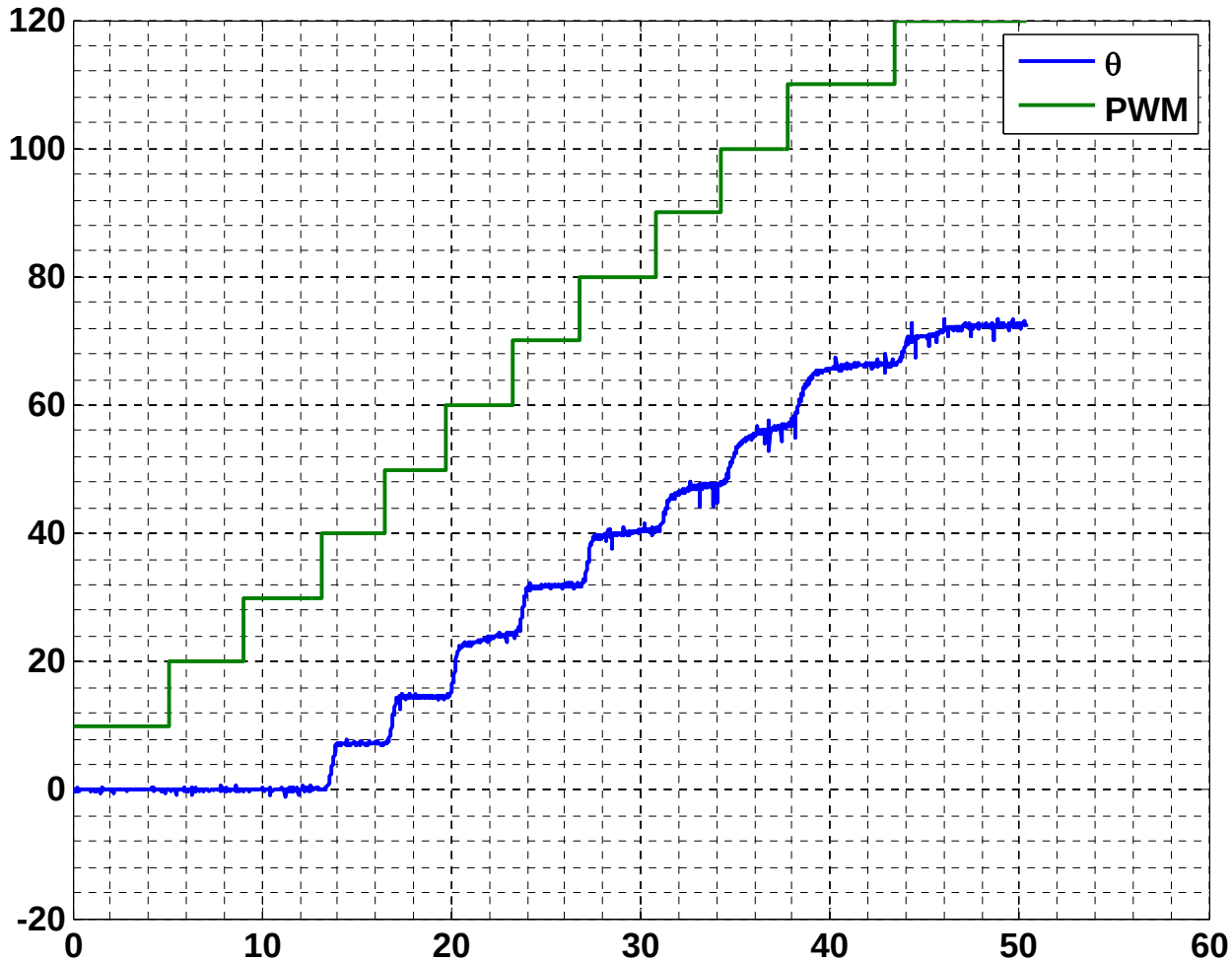
plot(t,theta,t,PWM); grid minor



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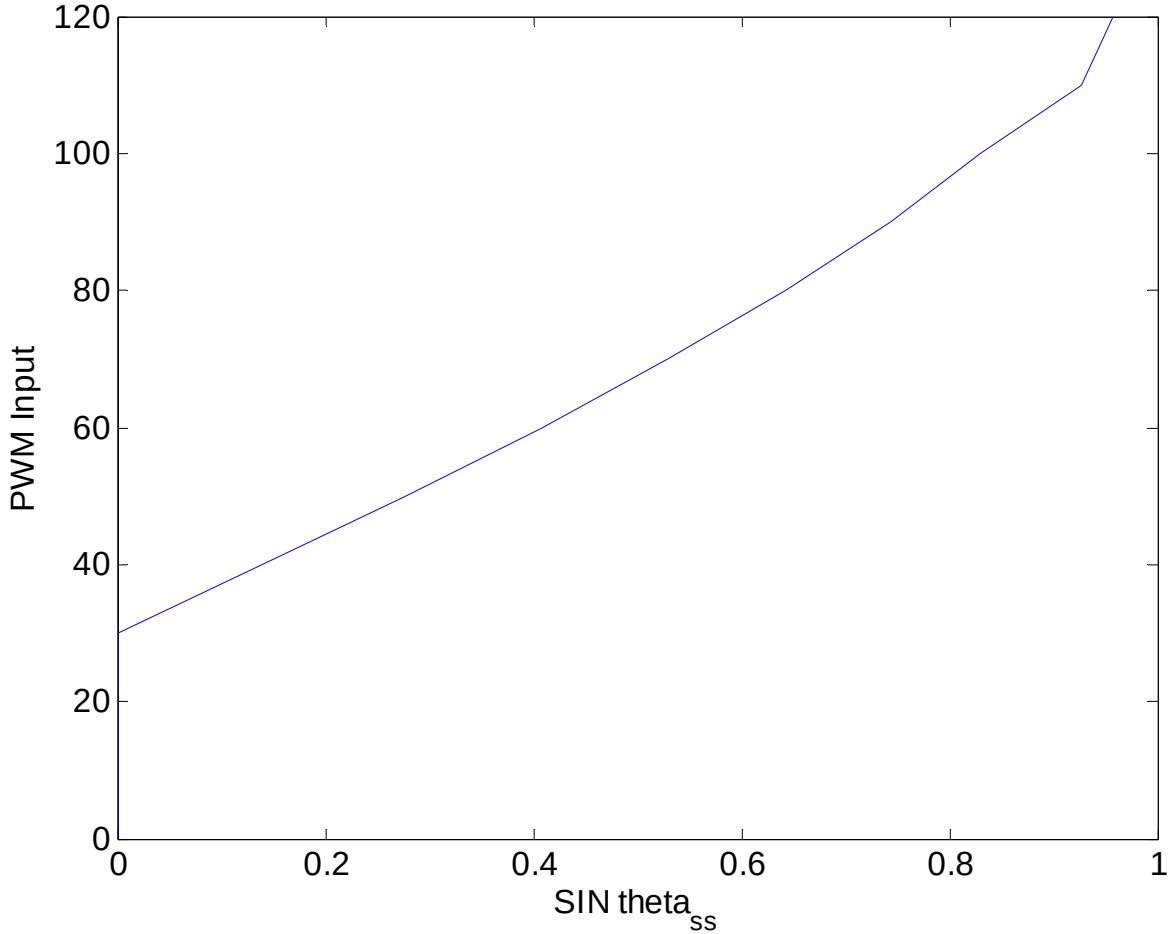
Gather Data

```
u_ss=[0 10 20 30 40 50 60 70 80 90 100
110 120];
theta_ss=[0 0 0 0 8 16 24 32 40 48 56 68
73];
sine_ss=sind(theta_ss);
plot(sine_ss,u_ss)
ylabel('PWM Input')
xlabel('SIN theta_{ss}')
```

Steady State Data



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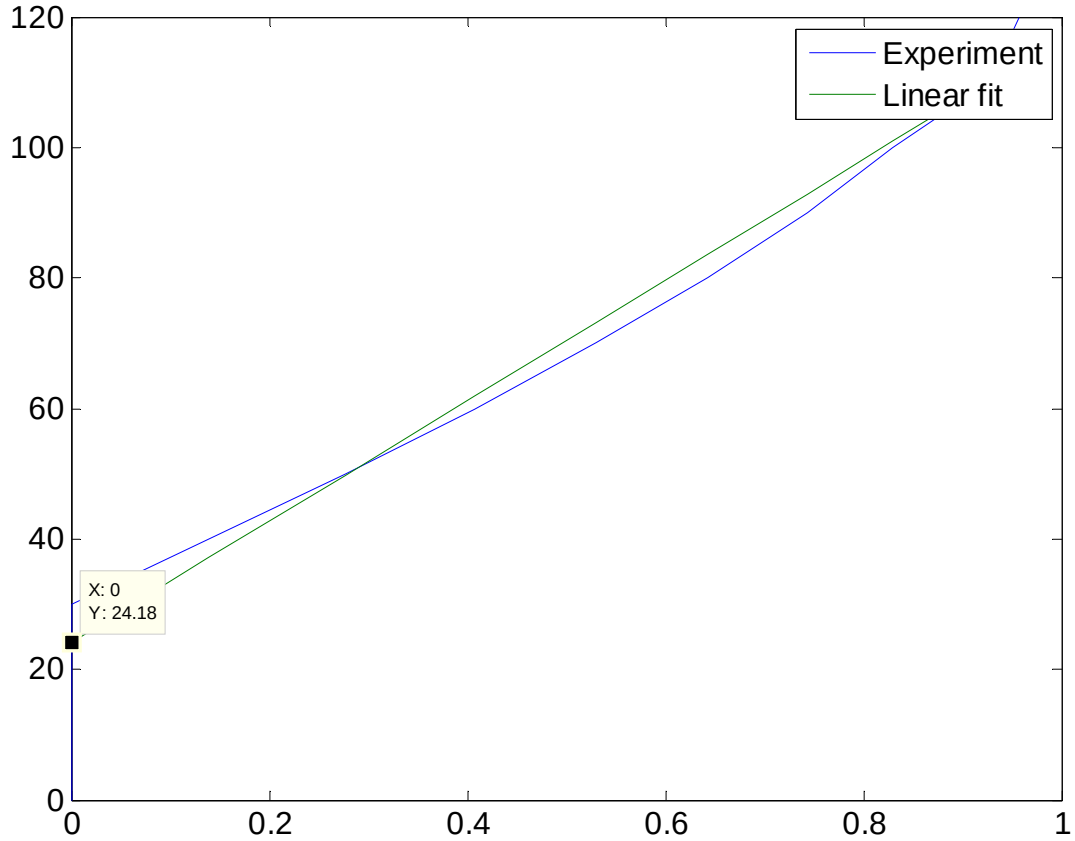
Fit a Line on Points 3-13

```
P=polyfit(sine_ss(3:13),u_ss(3:13),1) ;
plot(sine_ss,u_ss,sine_ss(3:13),
polyval(P,sine_ss(3:13)))
shg
legend('Experiment', 'Linear fit')
```

Slope and Offset

P = 92.4879 24.1771



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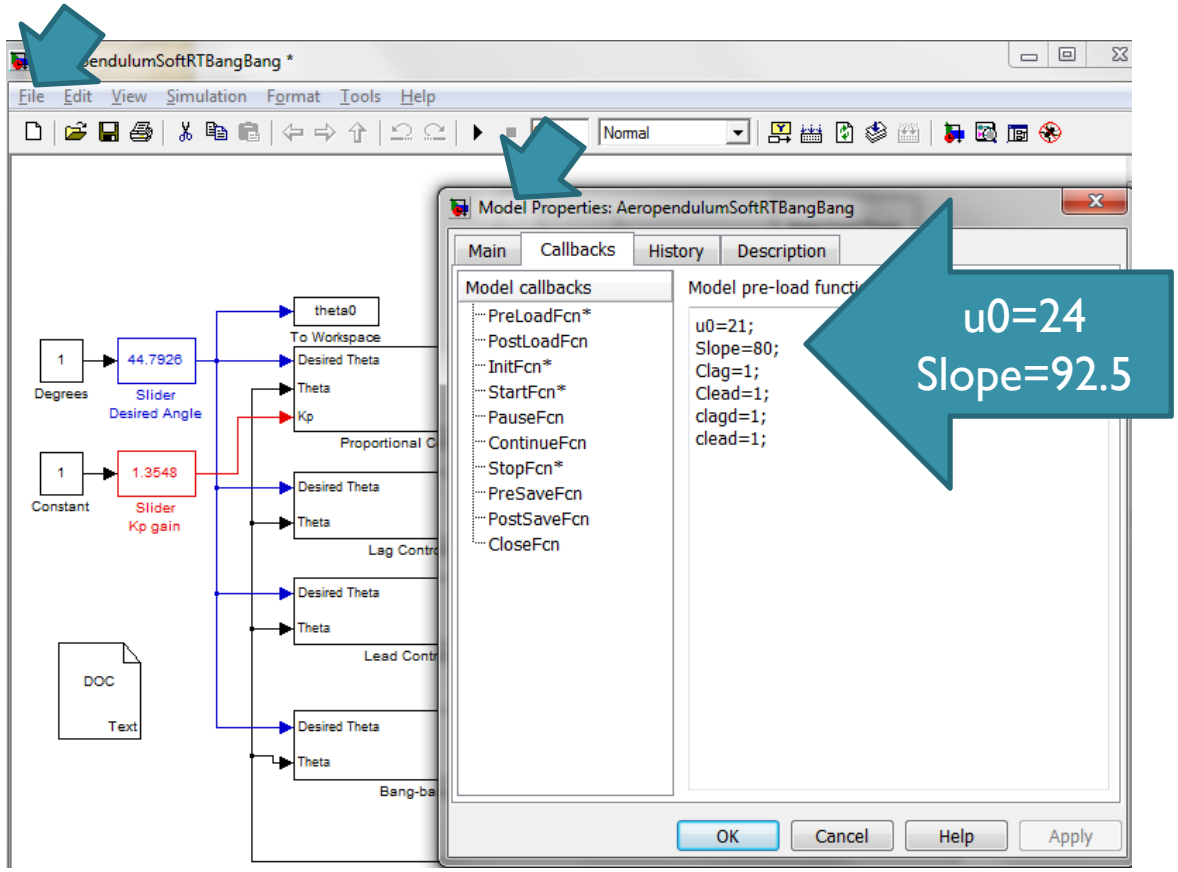
Project Installment # 2
Update Model

$$S = \frac{mg}{K} = 92.5$$

$$u_0 = 24$$



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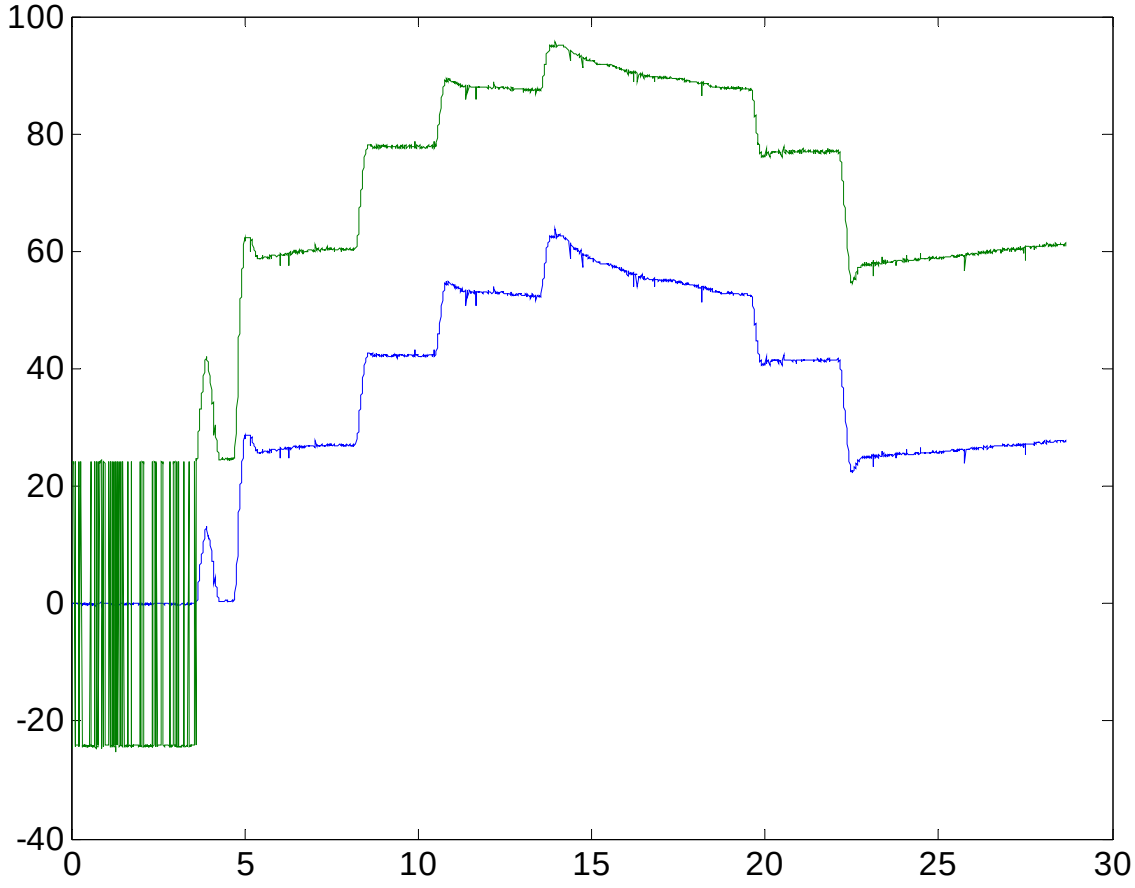
Test Using Closed Loop with Zero Gain (Slope needs adjustment, c. a 80)



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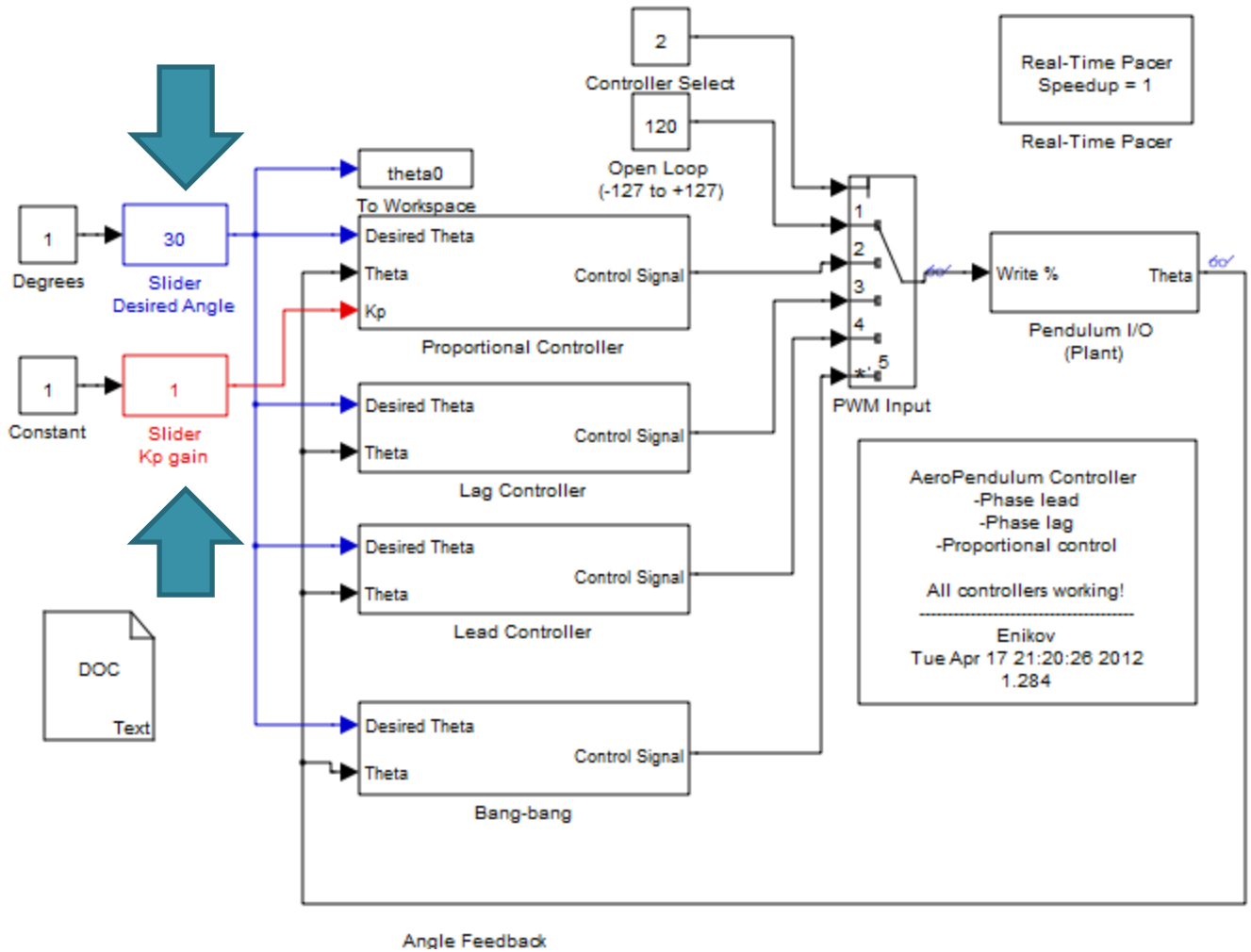
Check System Type using Kp=1 Theta=30



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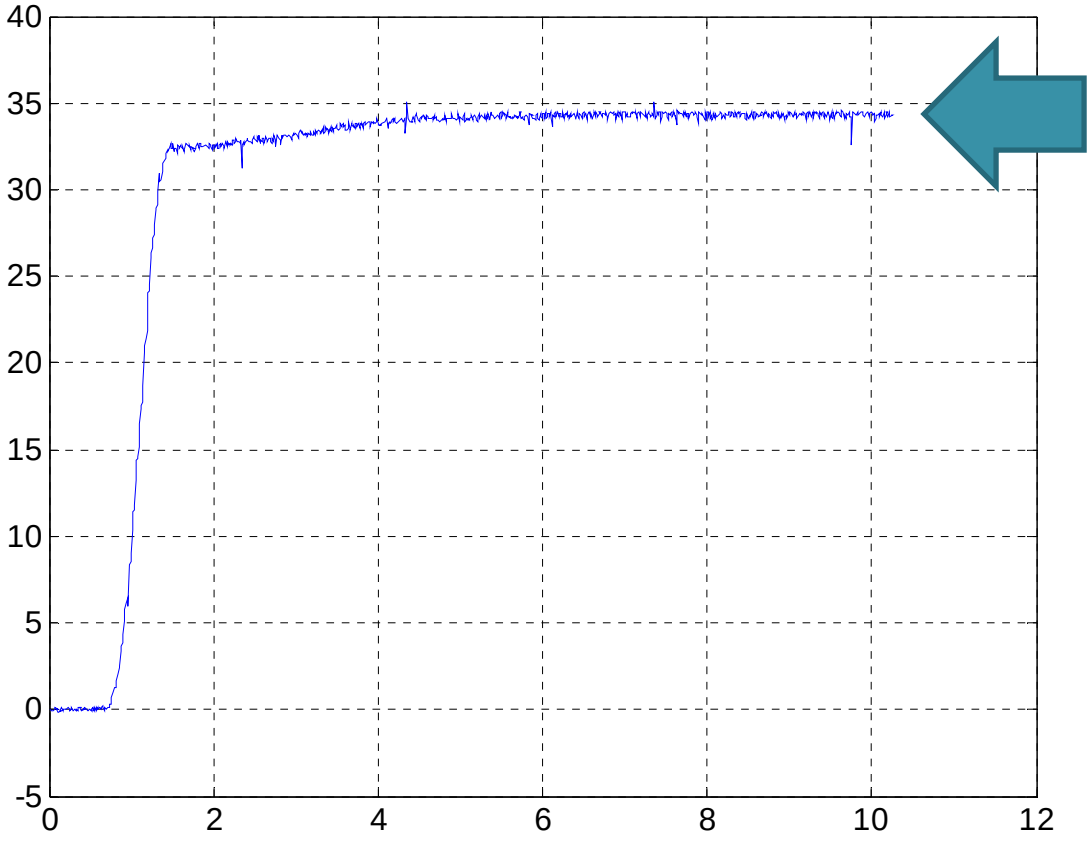
plot(t,theta)



4-5 degrees error (linearization Is not perfect)

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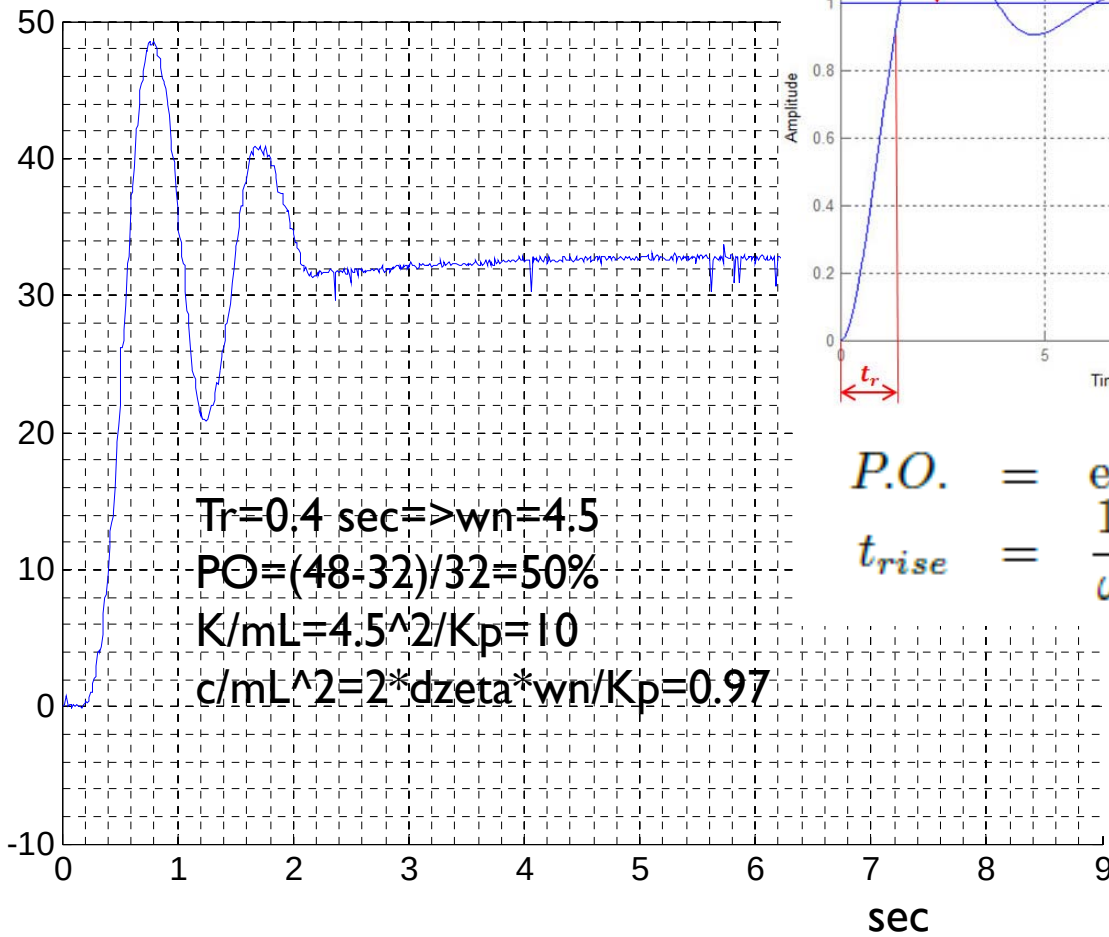
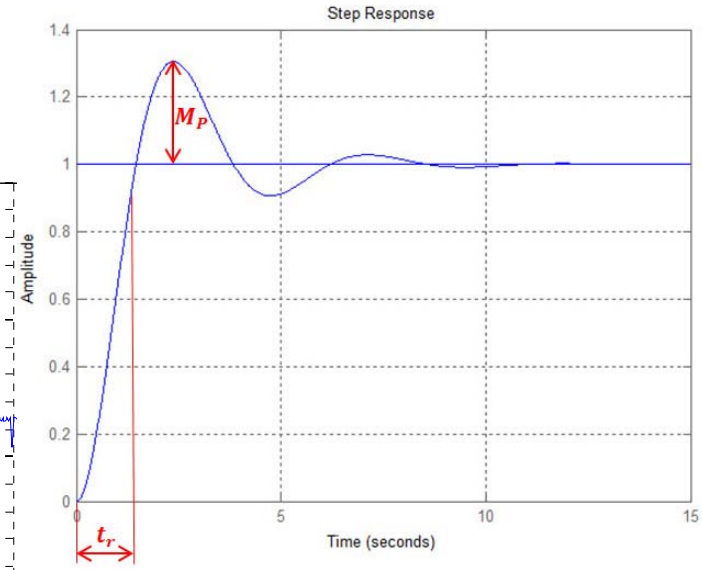
Identify Parameters

- From the response extract approximate values for and , then calculate and . (Use formulas for overshoot, peak time, rise time etc. to find and and then related to the physical parameters).
- The plot achieved for proportional controller may produce an over-damped relation in which case you will not be able to find out the parameters by using the above formulas.
- Just try increasing the proportional gain K_p to the point when you start getting an overshoot.

Using Kp=2





$$P.O. = \exp\left(-\zeta\pi/\sqrt{1-\zeta^2}\right)$$

$$t_{rise} = \frac{1.8}{\omega_n}.$$

$$\omega_n^2 = \frac{K}{mL}$$

$$2\zeta\omega_n = \frac{c}{mL^2}.$$

Matlab for Damping > zeta=fzero(@(x)exp(-pi*x/sqrt(1-x^2))-0.5,0)
zeta=0.22

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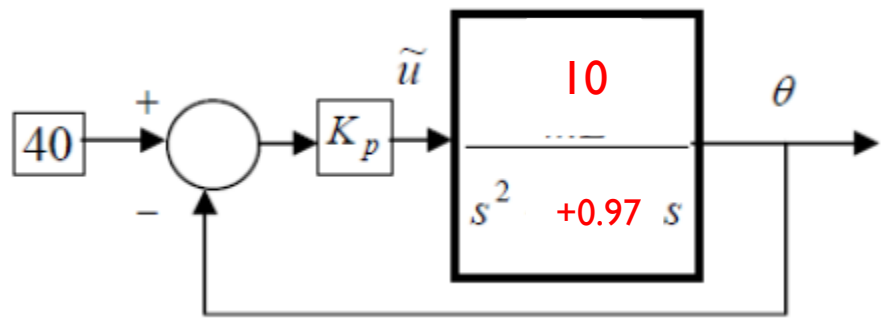
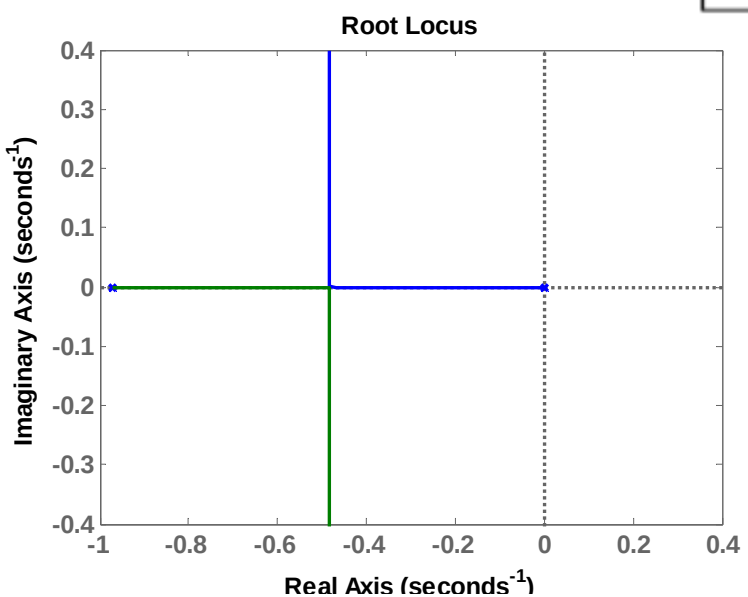
System Identified

- g=tf(10,[1 0.97 0])





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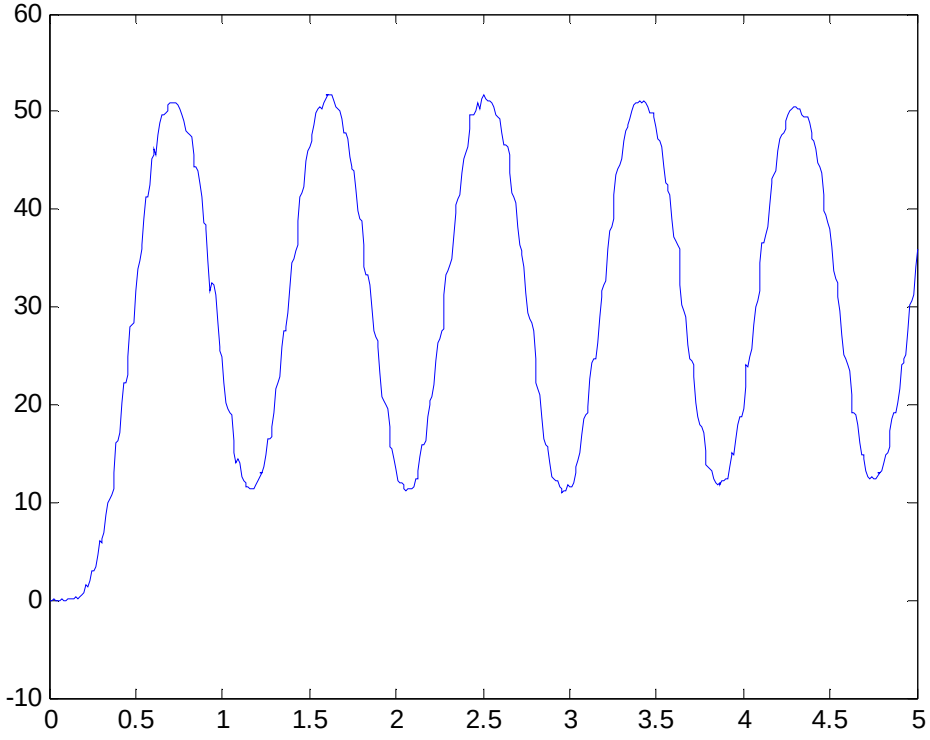
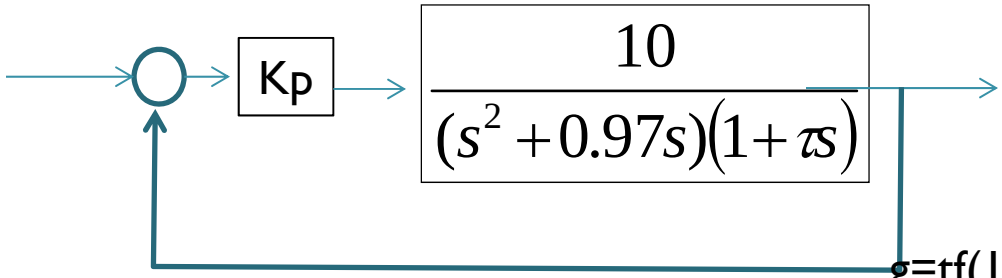
Test Stability with Kp=1, 2, 3…

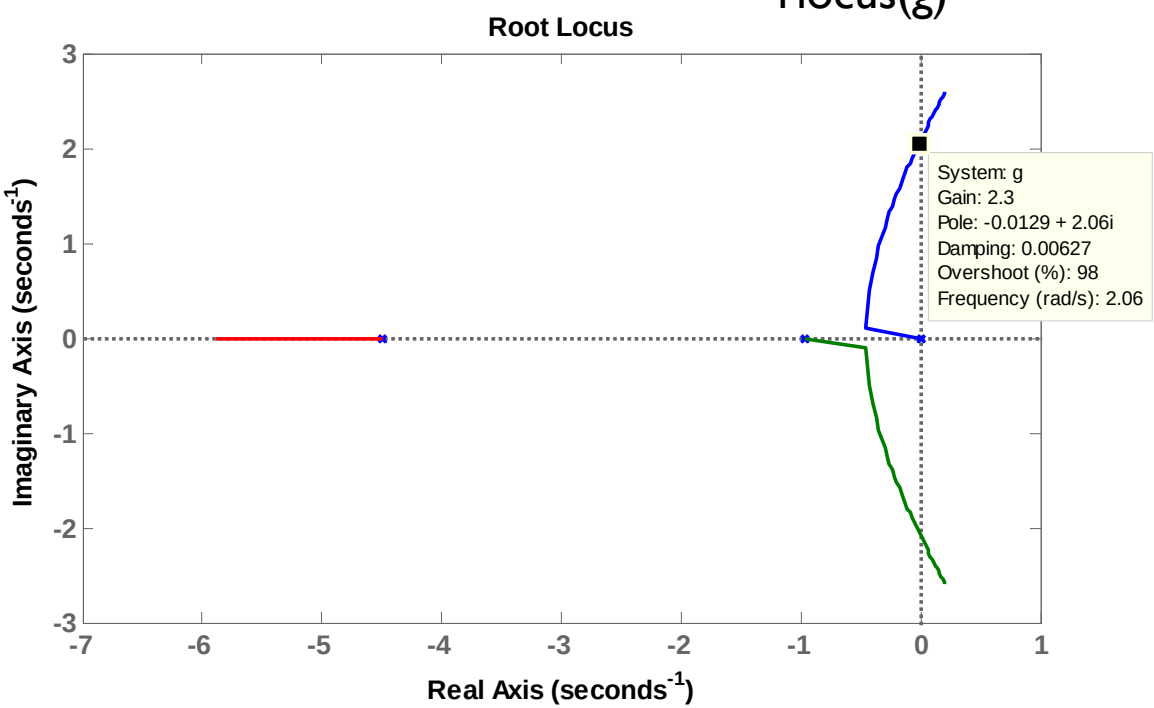
- Kp=2.4 ->stability limit



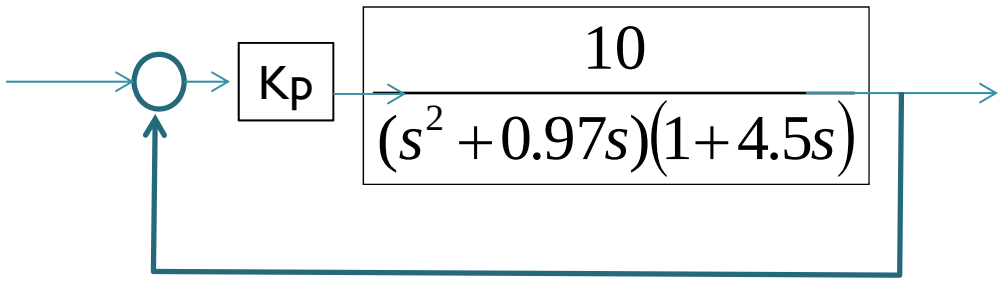
Modify The Model

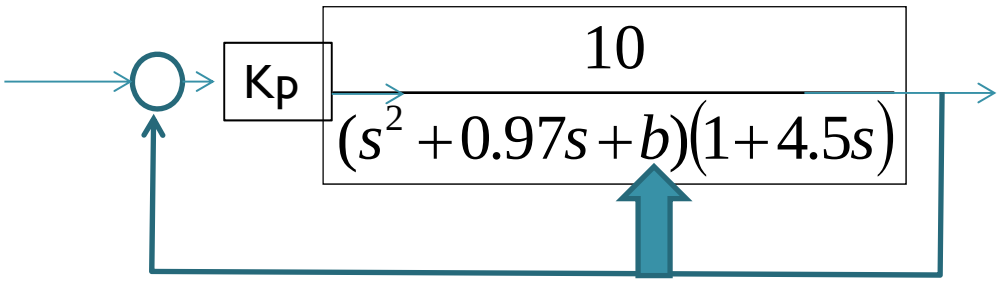



```
g=tf(10,conv([1 0.97 0],[1 4.5]))
rlocus(g)
```



Adjust for Steady State Error





$$e_{ss} = \frac{1}{1+\frac{10}{b}} = \frac{5}{30}$$

$$b = 2$$

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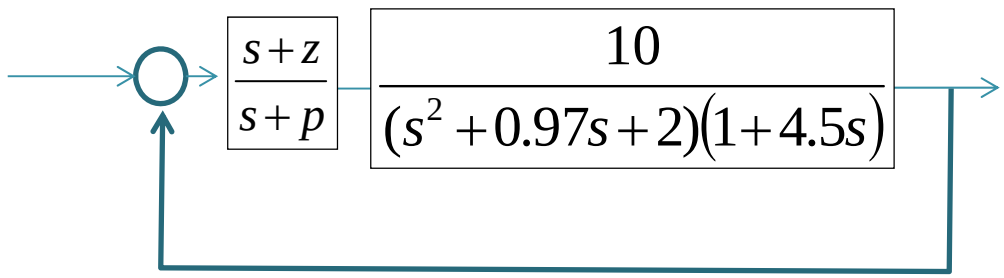
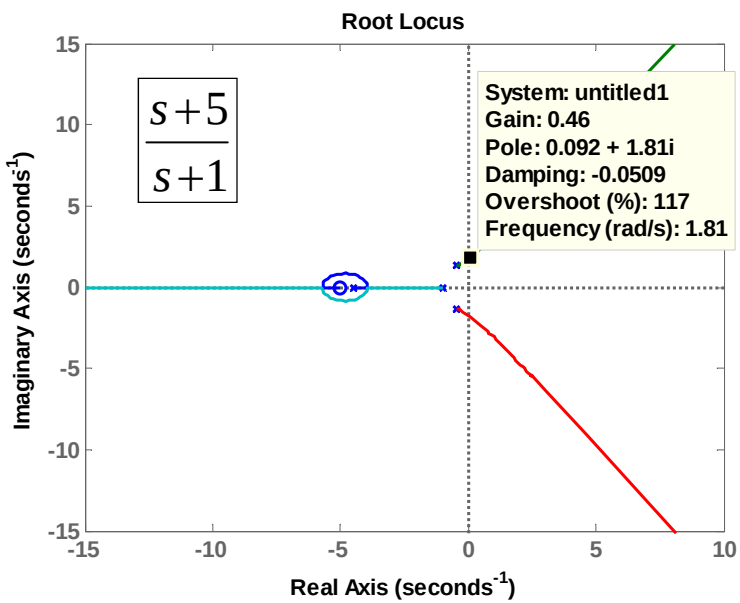
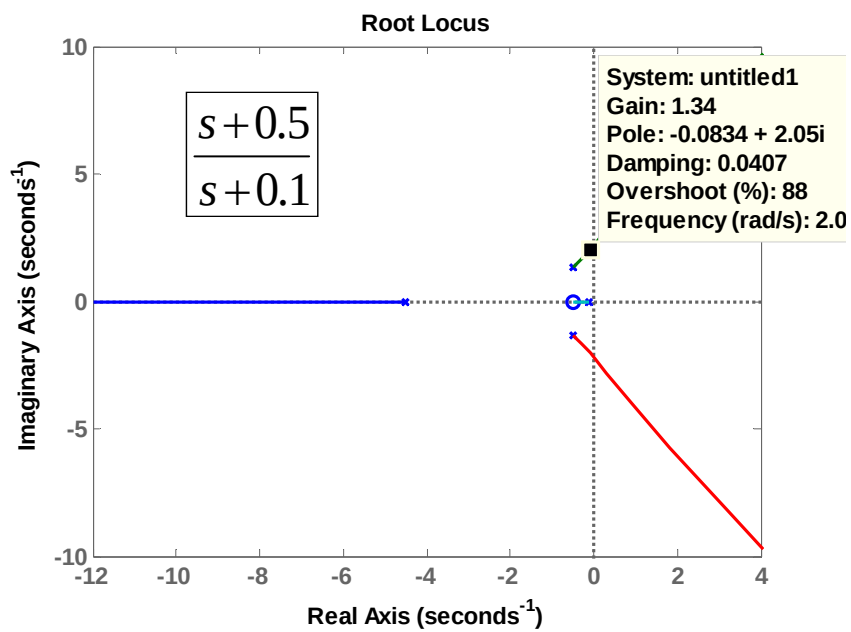
Lag Compensator







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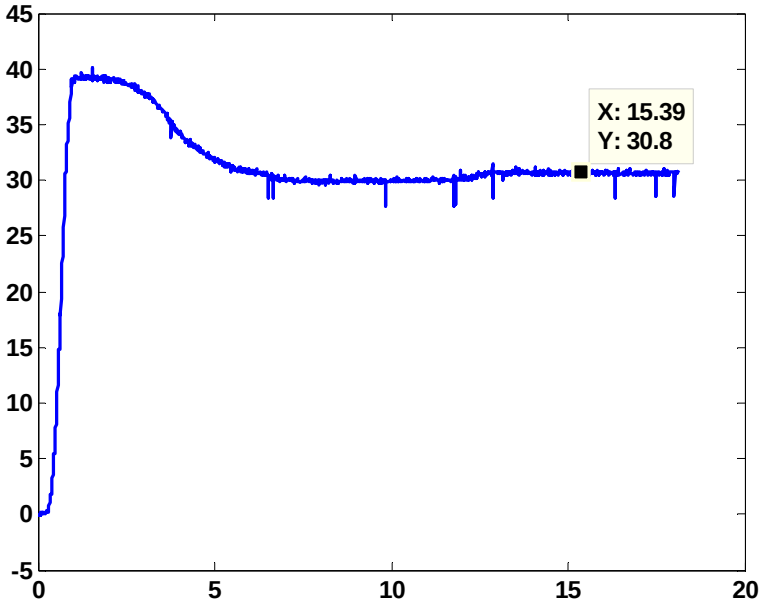
Step Response of Lag Compensation




```
c=tf([1 .5],[1 .1])
clagd=c2d(c,0.01)
```

Prof. Eke's Slides on Implementation

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Optional Activities

- Lead Compensator
- Lead-Lag Compensator
- On/Off Controller